٢٠١٢/٢/٩

الدوال العكسية

- Properties of Functions:

① $\begin{cases} y = 2x + 5 \longrightarrow \text{Relation} \\ x^2 + y^2 = 4 \longrightarrow \text{function} \end{cases}$

(each vertical line intersect with function in one point)
(each value of x have one value of y).

→ Is every function have inverse?
x is function of y (when each value of x has one value of y).
and it is converse

② ex
$\boxed{\begin{array}{l} y = x^2 \text{ (not inv.)} \\ y = 2x + 5 \text{ (inv.)} \end{array}}$
(each horizontal line intersect curve in one point).

→ one to one function. (the only function that has inverse).

(y is function of x)
(x is function of y)
} The condition one to one

if $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$
$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
} definition.

⇒ From (1) $2x_1 + 5 = 2x_2 + 5 \quad \therefore x_1 = x_2$ (one to one func)

⇒ From (2) $x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2 \quad \therefore x_1 \neq x_2$ (not one to one

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$y = x^2 \quad (x \geq 0) \Rightarrow$ is one to one when there is a condition in specific interval.

→ The relation between function and inverse?

$y = f(x) \rightarrow$ function

$x = f^{-1}(y) \rightarrow$ The Form of inverse function.

from (1) $f(x) = 2x + 5 \quad x = f^{-1}(y) = \dfrac{y - 5}{2}$

$f^{-1}(x) = \dfrac{x - 5}{2}$

→ The Relation between function and inverse? (f and f^{-1})

- Domain of f is Range of f^{-1}
- Range of f is domain of f^{-1}.
- graph is symmetric about ($y=x$)

$f^{-1}(f(x)) = x$, $f(f^{-1}(x)) = x$.

- inverse function: is a function of (one to one function)
  → if a function isn't one to one (There is an inverse function of specific interval).

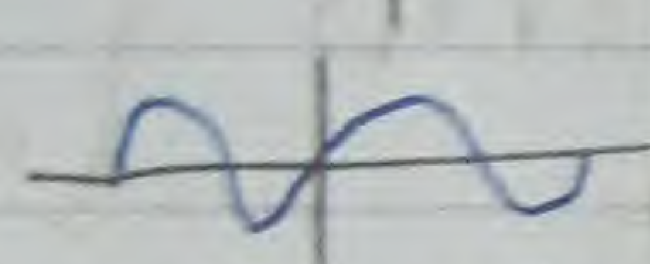
- Inversed Triagometric functions:

$y = \sin x$ → inverse : $y = \sin^{-1} x \neq \frac{1}{\sin x} = (\sin x)^{-1}$

Domain : R → $[-\pi/2, \frac{\pi}{2}]$ to be one to one function.

Range : $[-1, 1]$

Graph :

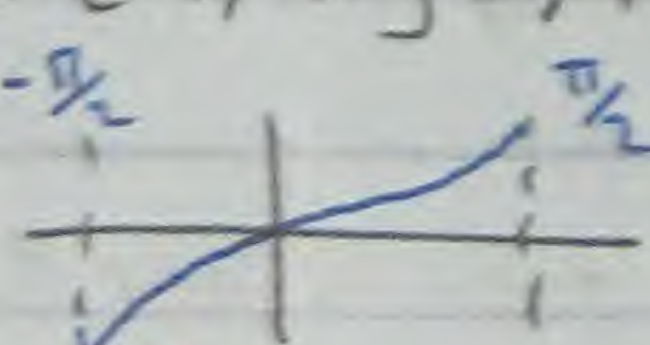


(not one to one)

Graph :

③ $y = \sin^{-1} x \Rightarrow D : [-1, 1]$, Range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

→ Graph:



→ Drevetive : $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

⇒ From ③

$\sin y = \sin(\sin^{-1} x) = x$

$x = \sin y$

$\frac{dx}{dy} = \cos y = \sqrt{1-\sin^2 y}$

$= \sqrt{1-x^2}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

Drevetive : $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

Note

$$\boxed{\frac{dy}{dx} = \left(\frac{1}{dx/dy}\right)}$$

$y = 2x+1$

$\frac{dy}{dx} = 2$

$x = \frac{y-1}{2}$

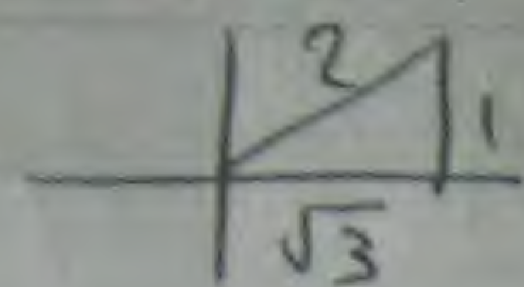
$\frac{dx}{dy} = \frac{1}{2}$

(complete drevetive)

$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$, if $\frac{d}{dx}(\sin^{-1}u(x)) = \frac{u'(x)}{\sqrt{1-u^2(x)}}$

→ $u'(x)$ → Drevetive of function but (u not u^2).

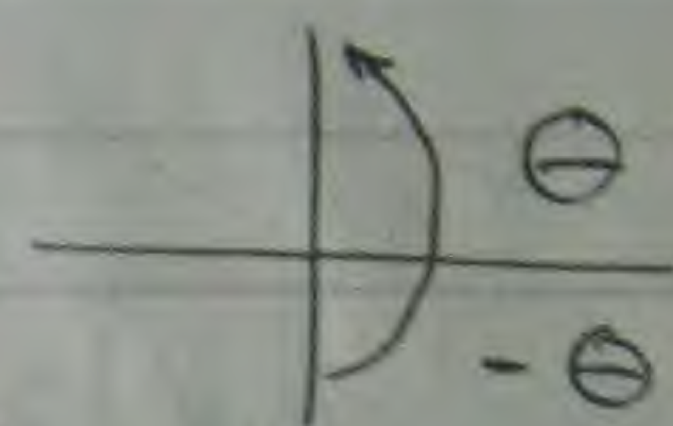
$\sin x = \frac{1}{2}$



$\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$ ⇒

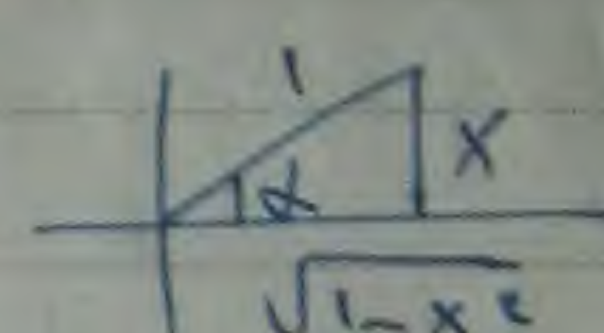
as Range of $\sin^{-1}x$ is $(\frac{-\pi}{2}, \frac{\pi}{2})$

$\sin^{-1}(\frac{-1}{2}) = -\frac{\pi}{6}$



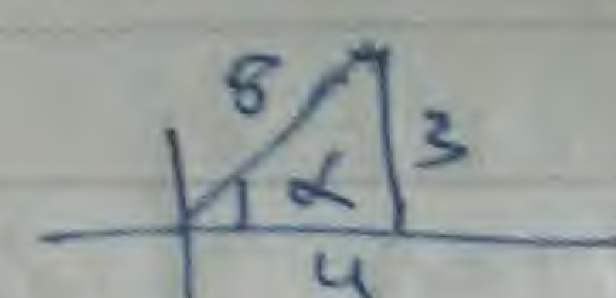
Note: each angle in the fourth quarter is (-ve).
as ~~Domain~~ Range $[-\frac{\pi}{2}, \frac{\pi}{2}]$. not $[0, 2\pi]$.

$\cos = (\sin^{-1}x) \Rightarrow \sin^{-1}x = \alpha$
$x = \sin\alpha.$



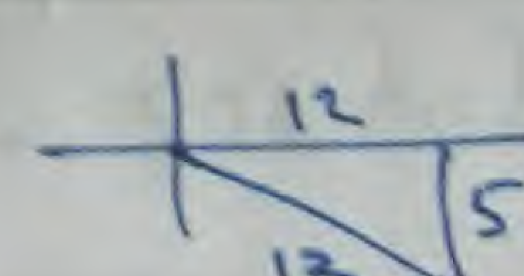
ex
⇒ $\tan(\underset{\alpha}{\sin^{-1}\frac{3}{5}} + \underset{\beta}{\sin^{-1}(-\frac{5}{13})})$

1) $\sin^{-1}\frac{3}{5} = \alpha$ $\sin\alpha = \frac{3}{5}$



$\tan\alpha = (\frac{3}{4})$

2) $\sin^{-1}(-\frac{5}{13}) = \beta$ $\sin\beta = \frac{-5}{13}$



$\tan\beta = (\frac{-5}{12})$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \frac{(\frac{3}{4}) + (\frac{-5}{12})}{1-(\frac{3}{4})(\frac{-5}{12})}.$$

ex
⇒ $y = \sin^{-1}(x^2)$

$$y' = \frac{2x}{\sqrt{1-(x^2)^2}}$$

$(x^2)^2$ ⇒ The function squared
$(2x)$ ⇒ derevitive of function not function's squared

ex ⇒ $y = \sin^{-1}(e^{2x})$.

$$y' = \frac{2e^{2x}}{\sqrt{1-(e^{2x})^2}}$$

ex ⇒ $y = \sin^{-1}(\sqrt{x^2+1})$

$$y' = \frac{\frac{2x}{2\sqrt{x^2+1}}}{\sqrt{1-(x^2+1)}}$$

ex $\Longrightarrow y = \sin^{-1}(\cos x)$.

$$y' = \frac{-\sin x}{\sqrt{1-\cos^2 x}} =$$

$$= \frac{-\sin x}{\sin x} = \boxed{-1}$$